Multiple i/p Multiple - o/p system - Discrete LTI

If a discrete-time LTI system has m inputs and p outputs and N state variables then state space representation:

$$q(n+1) = A\,q(n) + B\,x(n)$$

$$y(n) = C\,q(n) + D\,x(n)$$

where $q(n) = \begin{bmatrix} q_1(n) \\ q_2(n) \\ \vdots \\ q_N(n) \end{bmatrix}_{N}$, $x(n) = \begin{bmatrix} x_1(n) \\ x_2(n) \\ \vdots \\ x_m(n) \end{bmatrix}$.

$$y(n) = \begin{bmatrix} y_1(n) \\ y_2(n) \\ y_p(n) \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & \cdot & \cdot \\ \vdots & \cdot & \cdot \\ \cdot & & a_{NN} \end{bmatrix} \; N \times N \qquad B = \begin{bmatrix} b_{11} & \cdot & \cdot \\ & \cdot & b_{Nm} \\ \cdot & & \end{bmatrix} \; N \times m.$$

$$C = \begin{bmatrix} c_{11} & \cdot & \cdot \\ & & c_{pN} \\ \cdot & & \end{bmatrix}_{N \times p.} , \qquad D = \begin{bmatrix} d_{11} & \cdot & \cdot \\ d_{21} & & d_{pm} \\ \cdot & \cdot & \end{bmatrix} \; P \times m$$

State space representation of Discrete LTI system

⇒ The system is described by the difference eqn of Nth order.

(single i/p single o/p).

$$y(n) + a_1 y(n-1) + \dots a_N y(n-N) = x(n)$$

If $x(n)$ is given for $n \geq 0$, it requires N initial conditions $y(-1), y(-2), \dots y(-N)$ to determine the complete solution for $n > 0$

Let N state variables $q_1(n), q_2(n) \dots q_N(n)$.

$$\begin{cases} q_1(n) = y(n-N) \\ q_2(n) = y(n-(N-1)) = y(n-N+1) \\ \vdots \\ q_N(n) = y(N-1) \end{cases}$$

$$\therefore q_1(n+1) = q_2(n)$$

$$q_2(n+1) = q_3(n)$$

$$\vdots$$

$$q_N(n+1) = -a_N q_1(n) - a_{N-1} q_2(n) \dots - a_1 q_N(n) + x(n)$$

$$y(n) = -a_N q_1(n) - a_{N-1} q_2(n) \dots - a_1 q_N(n) + x(n)$$

$$\underbrace{\begin{bmatrix} q_1(n+1) \\ q_2(n+1) \\ q_N(n+1) \end{bmatrix}}_{\text{state vector}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ -a_N & -a_{N-1} & -a_{N-2} & \dots & -a_1 \end{bmatrix}}_{A \quad N \times N} \underbrace{\begin{bmatrix} q_1(n) \\ q_2(n) \\ q_N(n) \end{bmatrix}}_{q} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{b} x(n)$$

$$y(n) = \underbrace{\begin{bmatrix} -a_N & -a_{N-1} & \dots & -a_1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} q_1(n) \\ q_2(n) \\ \vdots \\ q_N(n) \end{bmatrix}}_{q(n)} + \underbrace{[1]}_{d} x(n)$$

$$\left.\begin{aligned} q(n+1) &= A q(n) + b x(n) \\ y(n) &= C q(n) + d x(n) \end{aligned}\right]$$

Scanned with CamScanner

by system matrix of the system difference eqn:

$y(n-2) + 2y(n-1) + 4y(n) = 0$

Sol^n second order differential eqn

$N = 2$, $q_1(n)$, $q_2(n)$

$q_1(n) = y(n-N) = y(n-2)$

$q_2(n) = y(n-N+1) = y(n-1)$

$\Rightarrow q_1(n+1) = q_2(n) = y(n-1)$

$\Rightarrow q_2(n+1) = -\frac{1}{4} q_1(n) - \cancel{2 q_2(n+1)} = y(n)$

and $q_2(n) = q_1(n+1)$.

$\therefore$ On substituting the state variables and ~~their~~ shifted version in the given system eqn:

$q_1(n) + 2 q_1(n+1) + 4 q_2(n+1) = 0$.

now $\therefore q_2(n+1) = -\frac{q_1(n)}{4} - \frac{2}{4} q_1(n+1)$.

$q_1(n+1) = q_2(n)$

$q_2(n+1) = -\frac{1}{4} q_1(n) - \frac{1}{2} q_1(n+1)$.

$= -\frac{1}{4} q_1(n) - \frac{1}{2} q_2(n)$

$$\begin{bmatrix} q_1(n+1) \\ q_2(n+2) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -\frac{1}{4} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} q_1(n) \\ q_2(n) \end{bmatrix} = AQ(n)$$

system A Matrix